Cuaderno de notas de trabajo

Carlos Graef Fernández

Cuaderno Sn 12

CAMPO CENTRAL.

SIMBOLOS DE BIRKHOFF DE PRIMERA ESPECIE.

Coordenadas Cartesianas.

$$a^i + \Delta^{im} B_{jk,m} \frac{dx^j}{ds} \frac{dx^k}{ds} = 0 ;$$

$$a_i + B_{jk,i} \frac{dx^j}{ds} \frac{dx^k}{ds} = 0 .$$

$$\boxed{B_{jk,i} = \frac{\partial h_{jk}}{\partial x^i} - \frac{\partial h_{ij}}{d x^k}}$$

$$\boxed{a_i = \left(\frac{\partial h_{ij}}{\partial x^k} - \frac{\partial h_{jk}}{\partial x^i} \right) \frac{dx^j}{ds} \frac{dx^k}{ds}}$$

$$\boxed{a_i = - B_{jk,i} \frac{dx^j}{ds} \frac{dx^k}{ds}}$$

$$\boxed{a_i = - B_{jk,i} v^j v^k}$$

$$\boxed{a_i + B_{jk,i} v^j v^k = 0}$$

SIMBOLOS DE BIRKHOFF DE PRIMERA

$B_{11,1}$	0	$B_{12,1}$	$+\frac{Mx}{r^3}$	$B_{13,1}$	$+\frac{My}{r^3}$	$B_{14,1}$	$+\frac{Mz}{r^3}$
$B_{21,1}$	0	$B_{22,1}$	0	$B_{23,1}$	0	$B_{24,1}$	0
$B_{31,1}$	0	$B_{32,1}$	0	$B_{33,1}$	0	$B_{34,1}$	0
$B_{41,1}$	0	$B_{42,1}$	0	$B_{43,1}$	0	$B_{44,1}$	0
$B_{11,2}$	$-\frac{Mx}{r^3}$	$B_{12,2}$	0	$B_{13,2}$	0	$B_{14,2}$	0
$B_{21,2}$	0	$B_{22,2}$	0	$B_{23,2}$	$+\frac{My}{r^3}$	$B_{24,2}$	$+\frac{Mz}{r^3}$
$B_{31,2}$	0	$B_{32,2}$	0	$B_{33,2}$	$-\frac{Mx}{r^3}$	$B_{34,2}$	0
$B_{41,2}$	0	$B_{42,2}$	0	$B_{43,2}$	0	$B_{44,2}$	$-\frac{Mx}{r^3}$

CENTRAL

ESPECIE. $B_{jk,i} = \frac{\partial h_{jk}}{\partial x^i} - \frac{\partial h_{ij}}{\partial x^k}$

$B_{11,3}$	$-\frac{My}{r^3}$	$B_{12,3}$	0	$B_{13,3}$	0	$B_{14,3}$	0
$B_{21,3}$	0	$B_{22,3}$	$-\frac{My}{r^3}$	$B_{23,3}$	0	$B_{24,3}$	0
$B_{31,3}$	0	$B_{32,3}$	$+\frac{Mx}{r^3}$	$B_{33,3}$	0	$B_{34,3}$	$+\frac{Mz}{r^3}$
$B_{41,3}$	0	$B_{42,3}$	0	$B_{43,3}$	0	$B_{44,3}$	$-\frac{My}{r^3}$
$B_{11,4}$	$-\frac{Mz}{r^3}$	$B_{12,4}$	0	$B_{13,4}$	0	$B_{14,4}$	0
$B_{21,4}$	0	$B_{22,4}$	$-\frac{Mz}{r^3}$	$B_{23,4}$	0	$B_{24,4}$	0
$B_{31,4}$	0	$B_{32,4}$	0	$B_{33,4}$	$-\frac{Mz}{r^3}$	$B_{34,4}$	0
$B_{41,4}$	0	$B_{42,4}$	$+\frac{Mx}{r^3}$	$B_{43,4}$	$+\frac{My}{r^3}$	$B_{44,4}$	0

Notación.

Tensores simétricos

$$B_{\underset{\smile}{jk},i} = \frac{1}{2} B_{jk,i} + \frac{1}{2} B_{kj,i} \;;$$

$$B_{\underset{\smile}{jk,i}} = \frac{1}{2} B_{jk,i} + \frac{1}{2} B_{ik,j} \;;$$

$$B_{j\underset{\smile}{k,i}} = \frac{1}{2} B_{jk,i} + \frac{1}{2} B_{ji,k} \,.$$

Tensores antisimétricos.

$$B_{\underset{\vee}{jk},i} = \frac{1}{2} B_{jk,i} - \frac{1}{2} B_{kj,i} \;;$$

$$B_{\underset{\vee}{jk,i}} = \frac{1}{2} B_{jk,i} - \frac{1}{2} B_{ik,j} \;;$$

$$B_{j\underset{\vee}{k,i}} = \frac{1}{2} B_{jk,i} - \frac{1}{2} B_{ji,k} \,.$$

CAMPO

SIMBOLOS DE BIRKHOFF DE PRIMERA ESPECIE.

$B_{11,1}$	0	$B_{12,1}$	$+\frac{Mx}{2r^3}$	$B_{13,1}$	$+\frac{My}{2r^3}$	$B_{14,1}$	$+\frac{Mz}{2r^3}$
$B_{21,1}$	$+\frac{Mx}{2r^3}$	$B_{22,1}$	0	$B_{23,1}$	0	$B_{24,1}$	0
$B_{31,1}$	$+\frac{My}{2r^3}$	$B_{32,1}$	0	$B_{33,1}$	0	$B_{34,1}$	0
$B_{41,1}$	$+\frac{Mz}{2r^3}$	$B_{42,1}$	0	$B_{43,1}$	0	$B_{44,1}$	0

$B_{11,2}$	$-\frac{Mx}{r^3}$	$B_{12,2}$	0	$B_{13,2}$	0	$B_{14,2}$	0
$B_{21,2}$	0	$B_{22,2}$	0	$B_{23,2}$	$+\frac{My}{2r^3}$	$B_{24,2}$	$+\frac{Mz}{2r^3}$
$B_{31,2}$	0	$B_{32,2}$	$+\frac{My}{2r^3}$	$B_{33,2}$	$-\frac{Mx}{r^3}$	$B_{34,2}$	0
$B_{41,2}$	0	$B_{42,2}$	$+\frac{Mz}{2r^3}$	$B_{43,2}$	0	$B_{44,2}$	$-\frac{Mx}{r^3}$

CENTRAL.

SIMETRICOS. $B_{jk,i} = \frac{\partial h_{jk}}{\partial x^i} - \frac{1}{2}\frac{\partial h_{ij}}{\partial x^k} - \frac{1}{2}\frac{\partial h_{ik}}{\partial x^j}$

$B_{11,3}$	$-\frac{My}{r^3}$	$B_{12,3}$	0	$B_{13,3}$	0	$B_{14,3}$	0
$B_{21,3}$	0	$B_{22,3}$	$-\frac{My}{r^3}$	$B_{23,3}$	$+\frac{Mx}{2r^3}$	$B_{24,3}$	0
$B_{31,3}$	0	$B_{32,3}$	$+\frac{Mx}{2r^3}$	$B_{33,3}$	0	$B_{34,3}$	$+\frac{Mz}{2r^3}$
$B_{41,3}$	0	$B_{42,3}$	0	$B_{43,3}$	$+\frac{Mz}{2r^3}$	$B_{44,3}$	$-\frac{My}{r^3}$
$B_{11,4}$	$-\frac{Mz}{r^3}$	$B_{12,4}$	0	$B_{13,4}$	0	$B_{14,4}$	0
$B_{21,4}$	0	$B_{22,4}$	$-\frac{Mz}{r^3}$	$B_{23,4}$	0	$B_{24,4}$	$+\frac{Mx}{2r^3}$
$B_{31,4}$	0	$B_{32,4}$	0	$B_{33,4}$	$-\frac{Mz}{r^3}$	$B_{34,4}$	$+\frac{My}{2r^3}$
$B_{41,4}$	0	$B_{42,4}$	$+\frac{Mx}{2r^3}$	$B_{43,4}$	$+\frac{My}{2r^3}$	$B_{44,4}$	0

SIMBOLOS DE BIRKHOFF DE PRIMERA ESPEC

$B_{11,1}$	0	$B_{12,1}$	$+\frac{Mx}{r^3}$	$B_{13,1}$	$+\frac{My}{r^3}$	$B_{14,1}$	$+\frac{Mz}{r^3}$
$B_{21,1}$	$-\frac{Mx}{2r^3}$	$B_{22,1}$	0	$B_{23,1}$	0	$B_{24,1}$	0
$B_{31,1}$	$-\frac{My}{2r^3}$	$B_{32,1}$	0	$B_{33,1}$	0	$B_{34,1}$	0
$B_{41,1}$	$-\frac{Mz}{2r^3}$	$B_{42,1}$	0	$B_{43,1}$	0	$B_{44,1}$	0

$B_{11,2}$	$-\frac{Mx}{2r^3}$	$B_{12,2}$	0	$B_{13,2}$	0	$B_{14,2}$	0
$B_{21,2}$	0	$B_{22,2}$	0	$B_{23,2}$	$+\frac{My}{r^3}$	$B_{24,2}$	$+\frac{Mz}{r^3}$
$B_{31,2}$	0	$B_{32,2}$	$-\frac{My}{2r^3}$	$B_{33,2}$	$-\frac{Mx}{2r^3}$	$B_{34,2}$	0
$B_{41,2}$	0	$B_{42,2}$	$-\frac{Mz}{2r^3}$	$B_{43,2}$	0	$B_{44,2}$	$-\frac{Mx}{2r^3}$

RAL

E. SIMETRICOS: $B_{jk,i} = \frac{1}{2}\frac{\partial h_{jk}}{\partial x^i} + \frac{1}{2}\frac{\partial h_{ik}}{\partial x^j} - \frac{\partial h_{ij}}{\partial x^k}$

$B_{11,3}$	$-\frac{My}{2r^3}$	$B_{12,3}$	0	$B_{13,3}$	0	$B_{14,3}$	0
$B_{21,3}$	0	$B_{22,3}$	$-\frac{My}{2r^3}$	$B_{23,3}$	$-\frac{Mx}{2r^3}$	$B_{24,3}$	0
$B_{31,3}$	0	$B_{32,3}$	$+\frac{Mx}{r^3}$	$B_{33,3}$	0	$B_{34,3}$	$+\frac{Mz}{r^3}$
$B_{41,3}$	0	$B_{42,3}$	0	$B_{43,3}$	$-\frac{Mz}{2r^3}$	$B_{44,3}$	$-\frac{My}{2r^3}$
$B_{11,4}$	$-\frac{Mz}{2r^3}$	$B_{12,4}$	0	$B_{13,4}$	0	$B_{14,4}$	0
$B_{21,4}$	0	$B_{22,4}$	$-\frac{Mz}{2r^3}$	$B_{23,4}$	0	$B_{24,4}$	$-\frac{Mx}{2r^3}$
$B_{31,4}$	0	$B_{32,4}$	0	$B_{33,4}$	$-\frac{Mz}{2r^3}$	$B_{34,4}$	$-\frac{My}{2r^3}$
$B_{41,4}$	0	$B_{42,4}$	$+\frac{Mx}{r^3}$	$B_{43,4}$	$+\frac{My}{r^3}$	$B_{44,4}$	0

CAMPO CENTRAL

SIMBOLOS DE BIRKHOFF DE PRIMERA ESPECIE

$B_{jk,}$

$B_{jk,i}$

B_{jk}

ECIE. SIMETRICOS. $B_{j\underbrace{k,i}} = 0$

$$B_{j\underbrace{k,i}} = \frac{1}{2} B_{jk,i} + \frac{1}{2} B_{ji,k}$$

$$B_{j\underbrace{k,i}} = \frac{1}{2} \frac{\partial h_{jk}}{\partial x^i} - \frac{1}{2} \frac{\partial h_{ji}}{\partial x^k} + \frac{1}{2} \frac{\partial h_{ji}}{\partial x^k} - \frac{1}{2} \frac{\partial h_{jk}}{\partial x^i}$$

$$\boxed{B_{j\underbrace{k,i}} = 0}$$

CAMPO CENTRAL.

SIMBOLOS DE BIRKHOFF DE PRIMERA ESPECIE.

ANTISIMÉTRICOS.

$$B_{\underset{\vee}{jk},i} = \frac{1}{2} B_{jk,i} - \frac{1}{2} B_{kj,i}$$

$$B_{\underset{\vee}{jk},i} = \cancel{\frac{1}{2} \frac{\partial h_{jk}}{\partial x^i}} - \frac{1}{2} \frac{\partial h_{ji}}{\partial x^k} - \cancel{\frac{1}{2} \frac{\partial h_{kj}}{\partial x^i}} + \frac{1}{2} \frac{\partial h_{ki}}{\partial x^j}$$

$$B_{\underset{\vee}{jk},i} = \frac{1}{2}\left(\frac{\partial h_{ki}}{\partial x^j} - \frac{\partial h_{ji}}{\partial x^k} \right)$$

$$B_{\underset{\vee}{jk},i} = \frac{1}{2}\left(\frac{\partial h_{ik}}{\partial x^j} - \frac{\partial h_{ij}}{\partial x^k} \right)$$

$$\boxed{B_{\underset{\vee}{jk},i} = \frac{1}{2} B_{ik,j}}$$

$$B_{\underset{\vee}{jk,i}} = \frac{1}{2} B_{jk,i} - \frac{1}{2} B_{ik,j}$$

$$B_{\underset{\vee}{jk,i}} = \frac{1}{2} \frac{\partial h_{jk}}{\partial x^i} - \cancel{\frac{1}{2} \frac{\partial h_{ji}}{\partial x^k}} - \frac{1}{2} \frac{\partial h_{ik}}{\partial x^j} + \cancel{\frac{1}{2} \frac{\partial h_{ij}}{\partial x^k}}$$

$$B_{\underset{\vee}{jk,i}} = \frac{1}{2}\left(\frac{\partial h_{jk}}{\partial x^i} - \frac{\partial h_{ik}}{\partial x^j} \right)$$

#

$$B_{jk,i} = \frac{1}{2}\left(\frac{\partial h_{kj}}{\partial x^i} - \frac{\partial h_{ki}}{\partial x^j}\right)$$

$$\boxed{B_{jk,i} = \frac{1}{2} B_{kj,i}}$$

$$B_{jk,i} = \frac{1}{2} B_{jk,i} - \frac{1}{2} B_{ji,k}$$

$$B_{jk,i} = \frac{1}{2}\frac{\partial h_{jk}}{\partial x^i} - \frac{1}{2}\frac{\partial h_{ji}}{\partial x^k} - \frac{1}{2}\frac{\partial h_{ji}}{\partial x^k} + \frac{1}{2}\frac{\partial h_{jk}}{\partial x^i}$$

$$B_{jk,i} = \left(\frac{\partial h_{jk}}{\partial x^i} - \frac{\partial h_{ji}}{\partial x^k}\right)$$

$$\boxed{B_{jk,i} = B_{jk,i}}$$

CAMPO CENTRAL

SIMBOLOS DE BIRKHOFF DE PRIMERA ESPECIE.

$B_{11,1}$	0	$B_{12,1}$	$+\frac{Mx}{2r^3}$	$B_{13,1}$	$+\frac{My}{2r^3}$	$B_{14,1}$	$+\frac{Mz}{2r^3}$
$B_{21,1}$	$-\frac{Mx}{2r^3}$	$B_{22,1}$	0	$B_{23,1}$	0	$B_{24,1}$	0
$B_{31,1}$	$-\frac{My}{2r^3}$	$B_{32,1}$	0	$B_{33,1}$	0	$B_{34,1}$	0
$B_{41,1}$	$-\frac{Mz}{2r^3}$	$B_{42,1}$	0	$B_{43,1}$	0	$B_{44,1}$	0
$B_{11,2}$	0	$B_{12,2}$	0	$B_{13,2}$	0	$B_{14,2}$	0
$B_{21,2}$	0	$B_{22,2}$	0	$B_{23,2}$	$+\frac{My}{2r^3}$	$B_{24,2}$	$+\frac{Mz}{2r^3}$
$B_{31,2}$	0	$B_{32,2}$	$-\frac{My}{2r^3}$	$B_{33,2}$	0	$B_{34,2}$	0
$B_{41,2}$	0	$B_{42,2}$	$-\frac{Mz}{2r^3}$	$B_{43,2}$	0	$B_{44,2}$	0

ANTISIMETRICOS. $B_{jk,i} = \frac{1}{2}\left(\frac{\partial h_{ik}}{\partial x^j} - \frac{\partial h_{ij}}{\partial x^k}\right) = -\frac{1}{2} B_{ik,j}$

$B_{11,3}$	0	$B_{12,3}$	0	$B_{13,3}$	0	$B_{14,3}$	0
$B_{21,3}$	0	$B_{22,3}$	0	$B_{23,3}$	$-\frac{Mx}{2r^3}$	$B_{24,3}$	0
$B_{31,3}$	0	$B_{32,3}$	$+\frac{Mx}{2r^3}$	$B_{33,3}$	0	$B_{34,3}$	$+\frac{Mz}{2r^3}$
$B_{41,3}$	0	$B_{42,3}$	0	$B_{43,3}$	$-\frac{Mz}{2r^3}$	$B_{44,3}$	0
$B_{11,4}$	0	$B_{12,4}$	0	$B_{13,4}$	0	$B_{14,4}$	0
$B_{21,4}$	0	$B_{22,4}$	0	$B_{23,4}$	0	$B_{24,4}$	$-\frac{Mx}{2r^3}$
$B_{31,4}$	0	$B_{32,4}$	0	$B_{33,4}$	0	$B_{34,4}$	$-\frac{My}{2r^3}$
$B_{41,4}$	0	$B_{42,4}$	$+\frac{Mx}{2r^3}$	$B_{43,4}$	$+\frac{My}{2r^3}$	$B_{44,4}$	0

CAMPO CENTRAL

SIMBOLOS DE BIRKHOFF DE PRIMERA ESPEC.

$B_{11,1}$	0	$B_{12,1}$	0	$B_{13,1}$	0	$B_{14,1}$	0
$B_{21,1}$	$+\frac{Mx}{2r^3}$	$B_{22,1}$	0	$B_{23,1}$	0	$B_{24,1}$	0
$B_{31,1}$	$+\frac{My}{2r^3}$	$B_{32,1}$	0	$B_{33,1}$	0	$B_{34,1}$	0
$B_{41,1}$	$+\frac{Mz}{2r^3}$	$B_{42,1}$	0	$B_{43,1}$	0	$B_{44,1}$	0
$B_{11,2}$	$-\frac{Mx}{2r^3}$	$B_{12,2}$	0	$B_{13,2}$	0	$B_{14,2}$	0
$B_{21,2}$	0	$B_{22,2}$	0	$B_{23,2}$	0	$B_{24,2}$	0
$B_{31,2}$	0	$B_{32,2}$	$+\frac{My}{2r^3}$	$B_{33,2}$	$-\frac{Mx}{2r^3}$	$B_{34,2}$	0
$B_{41,2}$	0	$B_{42,2}$	$+\frac{Mz}{2r^3}$	$B_{43,2}$	0	$B_{44,2}$	$-\frac{Mx}{2r^3}$

ANTISIMETRICOS. $B_{jk,i} = \frac{1}{2}B_{kj,i} = \frac{1}{2}\left(\frac{\partial h_{kj}}{\partial x^i} - \frac{\partial h_{ki}}{\partial x^j}\right)$

$B_{11,3}$	$-\frac{My}{2r^3}$	$B_{12,3}$	0	$B_{13,3}$	0	$B_{14,3}$	0
$B_{21,3}$	0	$B_{22,3}$	$-\frac{My}{2r^3}$	$B_{23,3}$	$+\frac{Mx}{2r^3}$	$B_{24,3}$	0
$B_{31,3}$	0	$B_{32,3}$	0	$B_{33,3}$	0	$B_{34,3}$	0
$B_{41,3}$	0	$B_{42,3}$	0	$B_{43,3}$	$+\frac{Mz}{2r^3}$	$B_{44,3}$	$-\frac{My}{2r^3}$
$B_{11,4}$	$-\frac{Mz}{2r^3}$	$B_{12,4}$	0	$B_{13,4}$	0	$B_{14,4}$	0
$B_{21,4}$	0	$B_{22,4}$	$-\frac{Mz}{2r^3}$	$B_{23,4}$	0	$B_{24,4}$	$+\frac{Mx}{2r^3}$
$B_{31,4}$	0	$B_{32,4}$	0	$B_{33,4}$	$-\frac{Mz}{2r^3}$	$B_{34,4}$	$+\frac{My}{2r^3}$
$B_{41,4}$	0	$B_{42,4}$	0	$B_{43,4}$	0	$B_{44,4}$	0

CAMPO CENTRA

SIMBOLOS DE BIRKHOFF DE PRIMERA ESPECIE.

$B_{11,1}$	0	$B_{12,1}$	$+\frac{Mx}{r^3}$	$B_{13,1}$	$+\frac{My}{r^3}$	$B_{14,1}$	$+\frac{Mz}{r^3}$
$B_{21,1}$	0	$B_{22,1}$	0	$B_{23,1}$	0	$B_{24,1}$	0
$B_{31,1}$	0	$B_{32,1}$	0	$B_{33,1}$	0	$B_{34,1}$	0
$B_{41,1}$	0	$B_{42,1}$	0	$B_{43,1}$	0	$B_{44,1}$	0
$B_{11,2}$	$-\frac{Mx}{r^3}$	$B_{12,2}$	0	$B_{13,2}$	0	$B_{14,2}$	0
$B_{21,2}$	0	$B_{22,2}$	0	$B_{23,2}$	$+\frac{My}{r^3}$	$B_{24,2}$	$+\frac{Mz}{r^3}$
$B_{31,2}$	0	$B_{32,2}$	0	$B_{33,2}$	$-\frac{Mx}{r^3}$	$B_{34,2}$	0
$B_{41,2}$	0	$B_{42,2}$	0	$B_{43,2}$	0	$B_{44,2}$	$-\frac{Mx}{r^3}$

ANTISIMETRICOS. $B_{jk,i} = B_{jk;i} = \left(\frac{\partial h_{jk}}{\partial x^i} - \frac{\partial h_{ji}}{\partial x^k}\right)$

$B_{11,3}$	$-\frac{My}{r^3}$	$B_{12,3}$	0	$B_{13,3}$	0	$B_{14,3}$	0
$B_{21,3}$	0	$B_{22,3}$	$-\frac{My}{r^3}$	$B_{23,3}$	0	$B_{24,3}$	0
$B_{31,3}$	0	$B_{32,3}$	$+\frac{Mx}{r^3}$	$B_{33,3}$	0	$B_{34,3}$	$+\frac{Mz}{r^3}$
$B_{41,3}$	0	$B_{42,3}$	0	$B_{43,3}$	0	$B_{44,3}$	$-\frac{My}{r^3}$
$B_{11,4}$	$-\frac{Mz}{r^3}$	$B_{12,4}$	0	$B_{13,4}$	0	$B_{14,4}$	0
$B_{21,4}$	0	$B_{22,4}$	$-\frac{Mz}{r^3}$	$B_{23,4}$	0	$B_{24,4}$	0
$B_{31,4}$	0	$B_{32,4}$	0	$B_{33,4}$	$-\frac{Mz}{r^3}$	$B_{34,4}$	0
$B_{41,4}$	0	$B_{42,4}$	$+\frac{Mx}{r^3}$	$B_{43,4}$	$+\frac{My}{r^3}$	$B_{44,4}$	0

Programma:

1) $B_{jk,i}\, v^j$;

2) $B_{jk,i}\, v^k$;

3) $B_{jk,i}\, v^i$.

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4) $B_{jk,i}\, v^j$;

5) $B_{jk,i}\, v^i$;

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6) $B_{jk,i}\, v^j$;

7) $B_{jk,i}\, v^k$;

~~4) $B_{jk,i} = 0$~~

8)

$\boxed{B_{jk,i}\, v^j}$ antisimétrico

0	$-\frac{Mx\,t'}{r^3}$	$-\frac{My\,t'}{r^3}$	$-\frac{Mz\,t'}{r^3}$
$+\frac{Mx\,t'}{r^3}$	0	$-\frac{M}{r^3}(yx'-xy')$	$-\frac{M}{r^3}(zx'-xz')$
$+\frac{My\,t'}{r^3}$	$+\frac{M}{r^3}(yx'-xy')$	0	$-\frac{M}{r^3}(zy'-yz')$
$+\frac{Mz\,t'}{r^3}$	$+\frac{M}{r^3}(zx'-xz')$	$+\frac{M}{r^3}(zy'-yz')$	0

$\boxed{B_{jk,i}\, v^k}$

$+\frac{Mr'}{r^2}$	$-\frac{Mxt'}{r^3}$	$-\frac{Myt'}{r^3}$	$-\frac{Mzt'}{r^3}$
0	$+\frac{M}{r^3}(yy'+zz')$	$-\frac{M}{r^3}yx'$	$-\frac{M}{r^3}zx'$
0	$-\frac{M}{r^3}xy'$	$+\frac{M}{r^3}(xx'+zz')$	$-\frac{M}{r^3}zy'$
0	$-\frac{M}{r^3}xz'$	$-\frac{M}{r^3}yz'$	$+\frac{M}{r^3}(xx'+yy')$

$$\boxed{B_{pq,r}\, v^q = - B_{pr,q}\, v^q}$$

$$\boxed{B_{jk,i}\, v^i}$$

$-\frac{Mr'}{r^2}$	$+\frac{Mxt'}{r^3}$	$+\frac{Myt'}{r^3}$	$+\frac{Mzt'}{r^3}$
0	$-\frac{M}{r^3}(yy'+zz')$	$+\frac{M}{r^3}yx'$	$+\frac{M}{r^3}zx'$
0	$+\frac{M}{r^3}(xy')$	$-\frac{M}{r^3}(xx'+zz')$	$+\frac{M}{r^3}zy'$
0	$+\frac{M}{r^3}xz'$	$+\frac{M}{r^3}yz'$	$-\frac{M}{r^3}(xx'+yy')$

$$\boxed{B_{pq,r}\, v^q = -B_{pr,q}\, v^q}$$

pág 23 → $\mathcal{S}_{ji} = \boxed{\mathcal{S}(B_{jk,i}v^k) = \frac{1}{2}B_{jk,i}v^k + \frac{1}{2}B_{ik,j}v^k}$

$+\frac{Mr'}{r^2}$	$-\frac{Mxt'}{2r^3}$	$-\frac{Myt'}{2r^3}$	$-\frac{Mzt'}{2r^3}$
$-\frac{Mxt'}{2r^3}$	$+\frac{M}{r^3}(yy'+zz')$	$-\frac{M}{2r^3}(xy'+yx')$	$-\frac{M}{2r^3}(xz'+zx')$
$-\frac{Myt'}{2r^3}$	$-\frac{M}{2r^3}(xy'+yx')$	$+\frac{M}{r^3}(xx'+zz')$	$-\frac{M}{2r^3}(yz'+zy')$
$-\frac{Mzt'}{2r^3}$	$-\frac{M}{2r^3}(xz'+zx')$	$-\frac{M}{2r^3}(yz'+zy')$	$+\frac{M}{r^3}(xx'+yy')$

La traza del tensor $\mathcal{S}_{ji}$.

$$\mathcal{T}(\mathcal{S}_{ji}) = \Delta^{ji}\mathcal{S}_{ji}$$

$$\mathcal{T}(\mathcal{S}_{ji}) = \frac{M}{r^3}\left[rr' - 2xx' - 2yy' - 2zz'\right]$$

$$\mathcal{T}(\mathcal{S}_{ji}) = \frac{-Mrr'}{r^3} = -\frac{Mr'}{r^2}$$

$$\boxed{\mathcal{T}(\mathcal{S}_{ji}) = -\frac{Mr'}{r^2}}$$

$$\boxed{\mathcal{A}\left(B_{jk,i}\,v^k\right) = \tfrac{1}{2} B_{jk,i}\,v^k - \tfrac{1}{2} B_{ik,j}\,v^k}$$

0	$-\dfrac{Mxt'}{2r^3}$	$-\dfrac{Myt'}{2r^3}$	$-\dfrac{Mzt'}{2r^3}$
$+\dfrac{Mxt'}{2r^3}$	0	$+\dfrac{M}{2r^3}(xy'-yx')$	$-\dfrac{M}{2r^3}(zx'-xz'$
$+\dfrac{Myt'}{2r^3}$	$-\dfrac{M}{2r^3}(xy'-yx')$	0	$+\dfrac{M}{2r^3}(yz'-zy'$
$+\dfrac{Mzt'}{2r^3}$	$+\dfrac{M}{2r^3}(zx'-xz')$	$-\dfrac{M}{2r^3}(yz'-zy')$	0

$$\cancel{\mathcal{A}\left(B_{jk,i}\,v^k\right) =}$$

$$\boxed{\mathcal{A}\left(B_{pq,r}\,v^q\right) = \tfrac{1}{2} B_{qp,r}\,v^q}$$

Tabla.

$$\boxed{\begin{aligned} B_{sp,q}\, v^s &= 2\mathcal{A}\left(B_{ps,q}\, v^s\right) \\ B_{ps,q}\, v^s &= \mathcal{S}\left(B_{ps,q}\, v^s\right) + \mathcal{A}\left(B_{ps,q}\, v^s\right) \\ B_{pq,s}\, v^s &= -\mathcal{S}\left(B_{ps,q}\, v^s\right) - \mathcal{A}\left(B_{ps,q}\, v^s\right) \end{aligned}}$$

$$\boxed{B_{jk,i}\, v^j}$$

$+\frac{Mr'}{2r^2}$	$-\frac{Mxt'}{r^3}$	$-\frac{Myt'}{r^3}$	$-\frac{Mzt'}{r^3}$
$+\frac{Mxt'}{2r^3}$	$+\frac{M}{2r^3}(yy'+zz')$	$-\frac{M}{2r^3}(2yx'-xy')$	$-\frac{M}{2r^3}(2zx'-xz')$
$+\frac{Myt'}{2r^3}$	$+\frac{M}{2r^3}(yx'-2xy')$	$+\frac{M}{2r^3}(xx'+zz')$	$-\frac{M}{2r^3}(2zy'-yz')$
$+\frac{Mzt'}{2r^3}$	$+\frac{M}{2r^3}(zx'-2xz')$	$+\frac{M}{2r^3}(zy'-2yz')$	$+\frac{M}{2r^3}(xx'+yy')$

$$B_{jk,i}\, v^j =$$

$$\boxed{B_{sp,q}\, v^s = \frac{1}{2}\mathcal{S}\left(B_{ps,q}\, v^s\right) + \frac{3}{2}\mathcal{A}\left(B_{ps,q}\, v^s\right)}$$

$$\boxed{B_{ps,q}\, v^s = \frac{1}{2}\mathcal{S}\left(B_{ps,q}\, v^s\right) + \frac{3}{2}\mathcal{A}\left(B_{ps,q}\, v^s\right)}$$

$\boxed{B_{jk,i}\, v^i}$

$-\frac{Mr'}{r^2}$	$+\frac{Mxt'}{2r^3}$	$+\frac{Myt'}{2r^3}$	$+\frac{Mzt'}{2r^3}$
$+\frac{Mxt'}{2r^3}$	$-\frac{M}{r^3}(yy'+zz')$	$+\frac{M}{2r^3}(yx'+xy')$	$+\frac{M}{2r^3}(zx'+xz')$
$+\frac{Myt'}{2r^3}$	$+\frac{M}{2r^3}(yx'+xy')$	$-\frac{M}{r^3}(xx'+zz')$	$+\frac{M}{2r^3}(zy'+yz')$
$+\frac{Mzt'}{2r^3}$	$+\frac{M}{2r^3}(zx'+xz')$	$-\frac{M}{2r^3}(zy'+yz')$	$-\frac{M}{r^3}(xx'+yy')$

$$\cancel{B_{ps,s}\, v^s = -\mathcal{S}(B_{pq,s}\, v^s)}$$

$$\boxed{B_{pq,s}\, v^s = -\mathcal{S}(B_{ps,q}\, v^s)}$$

$$\boxed{B_{sp,q}\, v^s = -\frac{1}{2}\mathcal{S}\left(B_{ps,q}\, v^s\right) + \frac{3}{2}\mathcal{A}\left(B_{ps,q}\, v^s\right)}$$

$$\boxed{B_{jk,i}\, v^j}$$

$-\frac{Mr'}{2r^2}$	$-\frac{Mxt'}{2r^3}$	$-\frac{Myt'}{2r^3}$	$-\frac{Mzt'}{2r^3}$
$+\frac{Mxt'}{r^3}$	$-\frac{M}{2r^3}(yy'+zz')$	$-\frac{M}{2r^3}(yx'-2xy')$	$-\frac{M}{2r^3}(zx'-2xz')$
$+\frac{Myt'}{r^3}$	$+\frac{M}{2r^3}(2yx'-xy')$	$-\frac{M}{2r^3}(xx'+zz')$	$-\frac{M}{2r^3}(zy'-2yz')$
$+\frac{Mzt'}{r^3}$	$+\frac{M}{2r^3}(2zx'-xz')$	$+\frac{M}{2r^3}(2zy'-yz')$	$-\frac{M}{2r^3}(xx'+yy')$

$$\boxed{-\frac{1}{2}\mathcal{S}(B_{ps,q}\, v^s) + \frac{3}{2}\sigma t(B_{ps,q}\, v^s)}$$

$-\frac{Mr'}{2r^2}$	$-\frac{Mxt'}{2r^3}$	$-\frac{Myt'}{2r^3}$	$-\frac{Mzt'}{2r^3}$
$+\frac{Mxt'}{r^3}$	$-\frac{M}{2r^3}(yy'+zz')$	$+\frac{M}{2r^3}(2xy'-yx')$	$+\frac{M}{2r^3}(2xz'-zx')$
$+\frac{Myt'}{r^3}$	$+\frac{M}{2r^3}(2yx'-xy')$	$-\frac{M}{2r^3}(xx'+zz')$	$+\frac{M}{2r^3}(2yz'-zy')$
$+\frac{Mzt'}{r^3}$	$+\frac{M}{2r^3}(2zx'-xz')$	$+\frac{M}{2r^3}(2zy'-yz')$	$-\frac{M}{2r^3}(xx'+yy')$

$$\boxed{B_{jk,i}\, v^k}$$

$+\frac{Mr'}{r^2}$	$-\frac{Mxt'}{2r^3}$	$-\frac{Myt'}{2r^3}$	$-\frac{Mzt'}{2r^3}$
$-\frac{Mxt'}{2r^3}$	$+\frac{M}{r^3}(yy'+zz')$	$-\frac{M}{2r^3}(yx'+xy')$	$-\frac{M}{2r^3}(zx'+xz')$
$-\frac{Myt'}{2r^3}$	$-\frac{M}{2r^3}(yx'+xy')$	$+\frac{M}{r^3}(xx'+zz')$	$-\frac{M}{2r^3}(zy'+yz')$
$-\frac{Mzt'}{2r^3}$	$-\frac{M}{2r^3}(zx'+xz')$	$-\frac{M}{2r^3}(zy'+yz')$	$+\frac{M}{r^3}(xx'+yy')$

$$\boxed{B_{ps,q}\, v^s = \mathcal{S}(B_{ps,q}\, v^s)}$$

$$\boxed{B_{pq,s}\, v^s = -\tfrac{1}{2}\mathcal{S}(B_{qs,p}\, v^s) + \tfrac{3}{2}\mathcal{A}(B_{qs,p}\, v^s)}$$

$$\boxed{B_{pq,s}\, v^s = -\tfrac{1}{2}\mathcal{S}(B_{ps,q}\, v^s) + \tfrac{3}{2}\mathcal{A}(B_{ps,q}\, v^s)}$$

Estas relaciones son válidas para todos los campos gravitacionales, no sólo para el campo central.

Ver páginas 43-51 de este cuaderno.

Definición:

$$\boxed{\mathcal{S}_{pq} = \mathcal{S}(B_{ps,q}\, v^s) = \tfrac{1}{2} B_{ps,q}\, v^s + \tfrac{1}{2} B_{qs,p}\, v^s}$$

$$\boxed{\mathcal{A}_{pq} = \mathcal{A}(B_{ps,q}\, v^s) = \tfrac{1}{2} B_{ps,q}\, v^s - \tfrac{1}{2} B_{qs,p}\, v^s}$$

$$B_{sp,q}\, v^s = 2\,\mathcal{A}_{pq}\,;$$

$$B_{ps,q}\, v^s = \mathcal{S}_{pq} + \mathcal{A}_{pq}\,;$$

$$B_{pq,s}\, v^s = -\mathcal{S}_{pq} - \mathcal{A}_{pq}\,;$$

$$B_{sp,q}\, v^s = \tfrac{1}{2}\mathcal{S}_{pq} + \tfrac{3}{2}\mathcal{A}_{pq}\,;$$

$$B_{ps,q}\, v^s = \tfrac{1}{2}\mathcal{S}_{pq} + \tfrac{3}{2}\mathcal{A}_{pq}\,;$$

$$B_{pq,s}\, v^s = -\mathcal{S}_{pq}\,;$$

$$B_{sp,q}\, v^s = -\tfrac{1}{2}\mathcal{S}_{pq} + \tfrac{3}{2}\mathcal{A}_{pq}\,;$$

$$B_{ps,q}\, v^s = \mathcal{S}_{pq}\,;$$

$$B_{pq,s}\, v^s = -\tfrac{1}{2}\mathcal{S}_{pq} - \tfrac{3}{2}\mathcal{A}_{pq}\,;$$

$$B_{sp,q}\, v^s = 0 ;$$

$$B_{ps,q}\, v^s = 0 ;$$

$$B_{pq,s}\, v^s = 0 ;$$

$$B_{sp,q}\, v^s = -\tfrac{1}{2}\mathcal{S}_{pq} + \tfrac{1}{2}\mathcal{A}_{pq} ;$$

$$B_{ps,q}\, v^s = +\tfrac{1}{2}\mathcal{S}_{pq} - \tfrac{1}{2}\mathcal{A}_{pq} ;$$

$$B_{pq,s}\, v^s = -\mathcal{A}_{pq} ;$$

$$B_{sp,q}\, v^s = \tfrac{1}{2}\mathcal{S}_{pq} + \tfrac{1}{2}\mathcal{A}_{pq} ;$$

$$B_{ps,q}\, v^s = +\mathcal{A}_{pq}$$

$$B_{pq,s}\, v^s = -\tfrac{1}{2}\mathcal{S}_{pq} + \tfrac{1}{2}\mathcal{A}_{pq} ;$$

$$B_{sp,q}\, v^s = 2\mathcal{A}_{pq} ;$$

$$B_{ps,q}\, v^s = \mathcal{S}_{pq} + \mathcal{A}_{pq} ;$$

$$B_{pq,s}\, v^s = -\mathcal{S}_{pq} - \mathcal{A}_{pq} .$$

$$\boxed{\mathcal{S}_{pq} v^q}$$

$$\mathcal{S}_{1q} v^q = + \frac{M r' t'}{2r^2};$$

$$\mathcal{S}_{2q} v^q = -\frac{Mx}{2r^3} - \frac{Mx}{r^3}(x'^2+y'^2+z'^2) + \frac{Mx'r'}{2r^2};$$

$$\mathcal{S}_{3q} v^q = -\frac{My}{2r^3} - \frac{My}{r^3}(x'^2+y'^2+z'^2) + \frac{My'r'}{2r^2},$$

$$\mathcal{S}_{4q} v^q = -\frac{Mz}{2r^3} - \frac{Mz}{r^3}(x'^2+y'^2+z'^2) + \frac{Mz'r'}{2r^2}.$$

$$\boxed{\boxed{\mathcal{S}_{pq} v^q = -\frac{1}{2} a_p}}$$

$$\boxed{\mathcal{A}_{pq} v^q}$$

$$\mathcal{A}_{1q} v^q = -\frac{M r' t'}{2r^2};$$

$$\mathcal{A}_{2q} v^q = +\frac{Mx}{2r^3} + \frac{Mx}{r^3}\{x'^2 + y'^2 + z'^2\} - \frac{Mx' r'}{2r^2};$$

$$\mathcal{A}_{3q} v^q = +\frac{My}{2r^3} + \frac{My}{r^3}\{x'^2 + y'^2 + z'^2\} - \frac{My' r'}{2r^2};$$

$$\mathcal{A}_{4q} v^q = +\frac{Mz}{2r^3} + \frac{Mz}{r^3}\{x'^2 + y'^2 + z'^2\} - \frac{Mz' r'}{2r^2}.$$

$$\boxed{\mathcal{A}_{pq} v^q = \frac{1}{2} a_p}$$

~~$B_{st,p} v^s v^t$~~ ~~B~~

Formas cuadráticas en las velocidades.

$$B_{ps,t}\, v^s v^t = 0;$$

$$B_{sp,t}\, v^s v^t = a_p;$$

$$B_{st,p}\, v^s v^t = -a_p;$$

$$B_{ps,t}\, v^s v^t = +\tfrac{1}{2} a_p;$$

$$B_{sp,t}\, v^s v^t = +\tfrac{1}{2} a_p;$$

$$B_{st,p}\, v^s v^t = -a_p;$$

$$B_{ps,t}\, v^s v^t = -\tfrac{1}{2} a_p;$$

$$B_{sp,t}\, v^s v^t = a_p;$$

$$B_{st,p}\, v^s v^t = -\tfrac{1}{2} a_p;$$

$$B_{ps,t}\, v^s v^t = 0;$$

$$B_{sp,t}\, v^s v^t = 0;$$

$$B_{st,p}\, v^s v^t = 0;$$

$$B_{ps,t}\, v^s v^t = -\tfrac{1}{2} a_p;$$

$$B_{sp,t}\, v^s v^t = +\tfrac{1}{2} a_p$$

$$B_{st,p}\, v^s v^t = 0;$$

#

Formas cuadráticas en las velocidades: continuación

B_p

$$\boxed{\begin{aligned}
&B_{ps,t}\, v^s v^t = \tfrac{1}{2} a_p ;\\
&B_{sp,t}\, v^s v^t = 0 ;\\
&B_{st,p}\, v^s v^t = -\tfrac{1}{2} a_p ;\\
&B_{p\,s,t}\, v^s v^t = 0 ;\\
&B_{s\,p,t}\, v^s v^t = a_p ;\\
&B_{s\,t,p}\, v^s v^t = - a_p .
\end{aligned}}$$

Investigar $\mathcal{S}_{pq}\, \dot{a}^q$ y $\mathcal{A}_{pq}\, a^q$ ¡ojo!

Tensores de rango 2 formados con la aceleración y la velocidad de una partícula exploradora en el campo central.

$$a_1 = -\frac{M r' t'}{r^2}$$

$$a_2 = -\frac{Mx}{r^3} + \frac{2Mx}{r^3} t'^2 - \frac{Mx' r'}{r^2}$$

$$a_3 = -\frac{My}{r^3} + \frac{2My}{r^3} t'^2 - \frac{My' r'}{r^2}$$

$$a_4 = -\frac{Mz}{r^3} + \frac{2Mz}{r^3} t'^2 - \frac{Mz' r'}{r^2}$$

$$v_1 = t'$$

$$v_2 = -x'$$

$$v_3 = -y'$$

$$v_4 = -z'$$

$$L_{ij} = a_i v_j$$

$$L_{ji} = a_j v_i$$

Traza de L_{ij}

$$\boxed{\mathcal{T}(L_{ij}) = \Delta^{ij} L_{ij}}$$

$$\mathcal{T}(L_{ij}) = -\frac{Mr'}{r^2}t'^2 - \frac{Mrr'}{r^3} + \frac{2Mrr'}{r^3}t'^2 - \frac{Mr'}{r^2}\left[t'^2 - 1\right] = 0$$

$$\boxed{L_{ij} = a_i v_j}$$

$$\boxed{L_{ij} = a_i v_j}$$

$-\frac{Mr'}{r^2}t'^2$	$+\frac{Mr'x'}{r^2}t'$	$+\frac{Mr'y'}{r^2}t'$	$+\frac{Mr'z'}{r^2}t'$
$-\frac{Mx}{r^3}t'$ $+\frac{2Mx}{r^3}t'^3$ $-\frac{Mr'x'}{r^2}t'$	$+\frac{Mxx'}{r^3}$ $-\frac{2Mxx'}{r^3}t'^2$ $+\frac{Mr'x'^2}{r^2}$	$+\frac{MXY'}{r^3}$ $-\frac{2MXY'}{r^3}t'^2$ $+\frac{Mr'x'y'}{r^2}$	$+\frac{MXZ'}{r^3}$ $-\frac{2MXZ'}{r^3}t'^2$ $+\frac{Mr'x'z'}{r^2}$
$-\frac{My}{r^3}t'$ $+\frac{2My}{r^3}t'^3$ $-\frac{Mr'y'}{r^2}t'$	$+\frac{MYX'}{r^3}$ $-\frac{2MYX'}{r^3}t'^2$ $+\frac{Mr'x'y'}{r^2}$	$+\frac{Myy'}{r^3}$ $-\frac{2Myy'}{r^3}t'^2$ $+\frac{Mr'y'^2}{r^2}$	$+\frac{MYZ'}{r^3}$ $-\frac{2MYZ'}{r^3}t'^2$ $+\frac{Mr'y'z'}{r^2}$
$-\frac{Mz}{r^3}t'$ $+\frac{2Mz}{r^3}t'^3$ $-\frac{Mr'z'}{r^2}t'$	$+\frac{MZX'}{r^3}$ $-\frac{2MZX'}{r^3}t'^2$ $+\frac{Mr'x'z'}{r^2}$	$+\frac{MZY'}{r^3}$ $-\frac{2MZY'}{r^3}t'^2$ $+\frac{Mr'y'z'}{r^2}$	$+\frac{Mzz'}{r^3}$ $-\frac{2Mzz'}{r^3}t'^2$ $+\frac{Mr'z'^2}{r^2}$

$$\boxed{L_{ji} = a_j v_i}$$

$-\frac{Mr'}{r^2}t'^2$	$-\frac{Mx}{r^3}t' + \frac{2Mx}{r^3}t'^3 - \frac{Mr'x'}{r^2}t'$	$-\frac{My}{r^3}t' + \frac{2My}{r^3}t'^3 - \frac{Mr'y'}{r^2}t'$	$-\frac{Mz}{r^3}t' + \frac{2Mz}{r^3}t'^3 - \frac{Mr'z'}{r^2}t'$
$+\frac{Mr'x'}{r^2}t'$	$+\frac{Mxx'}{r^3} - \frac{2Mxx'}{r^3}t'^2 + \frac{Mr'x'^2}{r^2}$	$+\frac{Myx'}{r^3} - \frac{2Myx'}{r^3}t'^2 + \frac{Mr'x'y'}{r^2}$	$+\frac{Mzx'}{r^3} - \frac{2Mzx'}{r^3}t'^2 + \frac{Mr'x'z'}{r^2}$
$+\frac{Mr'y'}{r^2}t'$	$+\frac{Mxy'}{r^3} - \frac{2Mxy'}{r^3}t'^2 + \frac{Mr'x'y'}{r^2}$	$+\frac{Myy'}{r^3} - \frac{2Myy'}{r^3}t'^2 + \frac{Mr'y'^2}{r^2}$	$+\frac{Mzy'}{r^3} - \frac{2Mzy'}{r^3}t'^2 + \frac{Mr'y'z'}{r^2}$
$+\frac{Mr'z'}{r^2}t'$	$+\frac{Mxz'}{r^3} - \frac{2Mxz'}{r^3}t'^2 + \frac{Mr'x'z'}{r^2}$	$+\frac{Myz'}{r^3} - \frac{2Myz'}{r^3}t'^2 + \frac{Mr'y'z'}{r^2}$	$+\frac{Mzz'}{r^3} - \frac{2Mzz'}{r^3}t'^2 + \frac{Mr'z'^2}{r^2}$

$$\boxed{L_{ij} = \frac{1}{2}\left(a_i v_j + a_j v_i\right)}$$

Simétrico

$-\frac{Mr'}{r^2}t'$	$-\frac{Mx}{2r^3}t' + \frac{Mx}{r^3}t'^3$	$-\frac{My}{2r^3}t' + \frac{My}{r^3}t'^3$	$-\frac{Mz}{2r^3}t' + \frac{Mz}{r^3}t'^3$
$-\frac{Mx}{2r^3}t' + \frac{Mx}{r^3}t'^3$	$+\frac{Mxx'}{r^3} - \frac{2Mxx'}{r^3}t'^2 + \frac{Mr'x'^2}{r^2}$	$+\frac{M}{2r^3}(xy'+yx') - \frac{M}{r^3}(xy'+yx')t'^2 + \frac{Mr'}{r^2}x'y'$	$+\frac{M}{2r^3}(xz'+zx') - \frac{M}{r^3}(xz'+zx')t'^2 + \frac{Mr'}{r^2}x'z'$
$-\frac{My}{2r^3}t' + \frac{My}{r^3}t'^3$	$+\frac{M}{2r^3}(xy'+yx') - \frac{M}{r^3}(xy'+yx')t'^2 + \frac{Mr'}{r^2}x'y'$	$+\frac{Myy'}{r^3} - \frac{2Myy'}{r^3}t'^2 + \frac{Mr'y'^2}{r^2}$	$+\frac{M}{2r^3}(yz'+zy') - \frac{M}{r^3}(yz'+zy') + \frac{Mr'}{r^2}y'z'$
$-\frac{Mz}{2r^3}t' + \frac{Mz}{r^3}t'^3$	$+\frac{M}{2r^3}(xz'+zx') - \frac{M}{r^3}(xz'+zx')t'^2 + \frac{Mr'x'z'}{r^2}$	$+\frac{M}{2r^3}(yz'+zy') - \frac{M}{r^3}(yz'+zy')t'^2 + \frac{Mr'y'z'}{r^2}$	$+\frac{Mzz'}{r^3} - \frac{2M}{r^3}zz't'^2 + \frac{Mr'z'^2}{r^2}$

$\boxed{\underset{V}{L_{ij}} = \frac{1}{2}L_{ij} - \frac{1}{2}L_{ij}}$ Antisimétrico.

0	$+\frac{Mx}{2r^3}t' - \frac{Mx}{r^3}t'^3 + \frac{Mr'x'}{r^2}t'$	$+\frac{My}{2r^3}t' - \frac{My}{r^3}t'^3 + \frac{Mr'y'}{r^2}t'$	$+\frac{Mz}{2r^3}t' - \frac{Mz}{r^3}t'^3 + \frac{Mr'z'}{r^2}t'$
$-\frac{Mx}{2r^3}t' + \frac{Mx}{r^3}t'^3 - \frac{Mr'x'}{r^2}t'$	0	$+\frac{M}{2r^3}(xy'-yx') - \frac{M}{r^3}(xy'-yx')t'^2$	$-\frac{M}{2r^3}(zx'-xz') + \frac{M}{r^3}(zx'-xz')t'^2$
$-\frac{My}{2r^3}t' + \frac{My}{r^3}t'^3 - \frac{Mr'y'}{r^2}t'$	$-\frac{M}{2r^3}(xy'-yx') + \frac{M}{r^3}(xy'-yx')t'^2$	0	$+\frac{M}{2r^3}(yz'-zy') - \frac{M}{r^3}(yz'-zy')t'^2$
$-\frac{Mz}{2r^3}t' + \frac{Mz}{r^3}t'^3 - \frac{Mr'z'}{r^2}t'$	$+\frac{M}{2r^3}(zx'-xz') - \frac{M}{r^3}(zx'-xz')t'^2$	$-\frac{M}{2r^3}(yz'-zy') + \frac{M}{r^3}(yz'-zy')t'^2$	0

$$\boxed{L_{ij} - A_{ij} + 2A_{ij}t'^2}$$

0	$+\frac{Mx}{r^3}t' - \frac{2Mx}{r^3}t'^3 + \frac{Mr'x'}{r^2}t'$	$+\frac{My}{r^3}t' - \frac{2My}{r^3}t'^3 + \frac{Mr'y'}{r^2}t'$	$+\frac{Mz}{r^3}t' - \frac{2Mz}{r^3}t'^3 + \frac{Mr'z'}{r^2}t'$
$-\frac{Mx}{r^3}t' + \frac{2Mx}{r^3}t'^3 - \frac{Mr'x'}{r^2}t'$	0	0	0
$-\frac{My}{r^3}t' + \frac{2My}{r^3}t'^3 - \frac{Mr'y'}{r^2}t'$	0	0	0
$-\frac{Mz}{r^3}t' + \frac{2Mz}{r^3}t'^3 - \frac{Mr'z'}{r^2}t'$	0	0	0

$$+\frac{Mx}{r^3}t' - \frac{2Mx}{r^3}t'^3 + \frac{Mrr'x'}{r^3}t' =$$

$$t'\left[\frac{M\cancel{x}}{r^3}\right]\left[\cancel{x} - 2xt'^2 + \{xx' + \cancel{y}y' + zz'\}x'\right] =$$

$$t'\left[\frac{M\cancel{x}}{r^3}\right]\left[xt'^2 - \cancel{xx'^2} - xy'^2 - xz'^2 - 2xt'^2 + \cancel{xx'^2} + yx'y' + zx'z'\right] =$$

$$\frac{Mt'}{r^3}\left[-xt'^2 - y'\{xy' - yx'\} - z'\{xz' - zx'\}\right]$$

$$\boxed{\frac{Mx}{r^3}t' - \frac{2Mx}{r^3}t'^3 + \frac{Mrr'x'}{r^3}t' =}$$

$$\boxed{-\frac{Mt'}{r^3}\left[+xt'^2 - y'\{xy' - yx'\} - z'\{xz' - zx'\}\right]}$$

$$L_{\underset{\smile}{ij}} = \frac{1}{2} a_i v_j + \frac{1}{2} a_j v_i$$

$$\boxed{L_{\underset{\vee}{ij}} = \frac{1}{2} a_i v_j - \frac{1}{2} a_j v_i}$$

$$L_{\underset{\vee}{ij}} - \mathcal{A}_{ij} + 2\mathcal{A}_{ij}\, t'^2 = \left(\overline{1 \; 0} \right)$$

$$\boxed{L_{\underset{\vee}{ij}} v^j = \frac{1}{2} a_i}$$

$$\boxed{\mathcal{A}_{ij} v^j = \frac{1}{2} a_i}$$

$$\frac{1}{2} a_i - \frac{1}{2} a_i + \frac{1}{2} a_i t'^2$$

$$\boxed{L_{\underset{\vee}{ij}} v^j - \mathcal{A}_{ij} v^j + 2\mathcal{A}_{ij} v^j = \frac{1}{2} a_i t'^2}$$

$$\boxed{\left[L_{ij} - \mathcal{A}_{ij} + 2\mathcal{A}_{ij} t'^2\right] v^j}$$

$$\boxed{W_i = \left[L_{ij} - \mathcal{A}_{ij} + 2\mathcal{A}_{ij} t'^2\right] v^j}$$

$$W_1 = + \frac{M}{r^3}\left[\{xx' + yy' + zz'\}t' - 2\{xx' + yy' + zz'\}t'^3 + rr'\{x'^2 + y'^2 + z'^2\}t'\right]$$

$$W_1 = \frac{M}{r^3}\left[rr't' - 2rr't'^3 + rr'\{t'^2 - 1\}t'\right]$$

$$W_1 = \frac{M}{r^3}\left[\cancel{rr't'} - 2rr't'^3 + rr't'^3 - \cancel{rr't'}\right]$$

$$W_1 = -\frac{M}{r^2}r't'^3 = a_1 t'^2$$

$$\boxed{W_1 = a_1 t'^2}$$

$$W_2 = t'^2\left[-\frac{Mx}{r^3} + \frac{2Mx}{r^3}t'^2 - \frac{Mr'x'}{r^2}\right]$$

$$\boxed{W_2 = a_2 t'^2}$$

$$\boxed{W_3 = a_3 t'^2}$$

$$\boxed{W_4 \doteq a_4 t'^2}$$

$$L_{ij} = \frac{1}{2} a_i v_j - \frac{1}{2} a_j v_i$$

$$L_{ij} - a_i \tau_j + a_j \tau_i = A_{ij} - 2A_{ij} t'^2$$

Multiplíquese por v^j

$$\cancel{\tfrac{1}{2} a_i} - a_i \tau_j v^j = \cancel{\tfrac{1}{2} a_i} - a_i t'^2$$ ¡Correcto!

$$\frac{1}{2} a_i v_j - \frac{1}{2} a_j v_i - a_i \tau_j + a_j \tau_i$$

$$a_i \left(\tfrac{1}{2} v_j - \tau_j\right) = a_j \left(\tfrac{1}{2} v_i - \tau_i\right)$$

$$\cancel{\frac{d}{ds}\left(\tfrac{1}{2} v_j - \tau_j\right) = \tfrac{1}{2} a_j - t''_j}$$

$$\cancel{\left(\tfrac{1}{2} a_i - t''\right)\left(\tfrac{1}{2} v_j - \tau_j\right)}$$

$$\boxed{\underset{\vee}{L_{ij}} - \mathcal{A}_{ij} + 2\mathcal{A}_{ij}\, t'^2}$$

0	$-a_2 v_1$	$-a_3 v_1$	$-a_4 v_1$
$+a_2 v_1$	0	0	0
$+a_3 v_1$	0	0	0
$+a_4 v_1$	0	0	0

$\underset{\vee}{L_{ij}} - \mathcal{A}_{ij} + 2\mathcal{A}_{ij}\, t'^2$ parece ser el tensor antisimétrico formado por los dos cuadrivectores:

$$\boxed{\hat{\tau} = (t', 0, 0, 0)}$$

$$\hat{a} = (a_1, a_2, a_3, a_4)$$

compa. covariantes

$$\leftarrow \boxed{\tau_i a_j - \tau_j a_i}$$

Consideremos un cuadrivector

$$x^i = (x^1, x^2, x^3, x^4)$$

$\Delta_{ij} x^i x^j$ es un invariante en las transformaciones de Lorentz.

$$(x^1)^2 - (x^2)^2 - (x^3)^2 - (x^4)^2 = \text{Invar.}$$

Si hacemos

$$x^1 = r, \quad x^2 = x, \quad x^3 = y, \quad x^4 = z$$

$$\Longrightarrow r^2 - x^2 - y^2 - z^2 = 0.$$

Invariante.

$$\left(r = \sqrt{x^2+y^2+z^2},\ x,\ y,\ z\right)$$

son las componentes de un cuadrivector $\hat{x}$.

Calcúlense las componentes del cuadrivector

$$\hat{\alpha} = \frac{d^2 \hat{x}}{ds^2}$$

de una partícula en el campo central.

$$\frac{d\mathcal{X}^i}{ds} = \mathcal{X}'^i$$

$$r^2 = x^2 + y^2 + z^2,$$

$$r r' = x x' + y y' + z z',$$

$$r' = \frac{x x' + y y' + z z'}{r}$$

$$r'' = \left[-\frac{r'}{r^2}\left(x x' + y y' + z z'\right) + \frac{1}{r}\left\{x x'' + y y'' + z z''\right\} + \frac{1}{r}\left\{x'^2 + y'^2 + z'^2\right\} \right]$$

$$x x'' = +\frac{Mx^2}{r^3} - \frac{2Mx^2}{r^3}t'^2 + \frac{Mr'(xx')}{r^2};$$

$$y y'' = +\frac{My^2}{r^3} - \frac{2My^2}{r^3}t'^2 + \frac{Mr'(yy')}{r^2};$$

$$z z'' = +\frac{Mz^2}{r^3} - \frac{2Mz^2}{r^3}t'^2 + \frac{Mr'(zz')}{r^2}.$$

$$x x'' + y y'' + z z'' = +\frac{M}{r} - \frac{2M}{r}t'^2 + \frac{M r'^2}{r}.$$

$$r'' = \left[-\frac{r'^2}{r} + \frac{M}{r^2} - \frac{2M}{r^2} t'^2 + \frac{M r'^2}{r^2} + \frac{1}{r}\{t'^2 - 1\} \right]$$

Símbolos de Birkhoff de Primera Especie!

$$B_{ijk} = \frac{\partial h_{ij}}{\partial x^k} - \frac{\partial h_{ik}}{\partial x^j}$$

$$\boxed{\frac{\partial h_{ij}}{\partial x^k} = Z_{ijk}}$$

El tensor Z_{ijk} es simétrico en i, j

$$B_{ijk} = Z_{ijk} - Z_{ikj} \;;$$

$$B_{ij,k} = Z_{ijk} - Z_{ikj} \;;$$

$$B_{ij,k} = Z_{ijk} - Z_{ikj}$$

$$B_{ijk} = Z_{ijk} - Z_{ikj} = 0$$

$$B_{ij,k} = Z_{ijk} - Z_{ikj}$$

$$B_{ij,k} = Z_{ij,k} - Z_{ikj}$$

$$B_{ijk} = Z_{ijk} - Z_{ikj}$$

$$B_{ij,k} = Z_{ijk} - Z_{ikj} \; ;$$

$$B_{ij,k} = Z_{ijk} - \tfrac{1}{2} Z_{ikj} - \tfrac{1}{2} Z_{jki} \; ;$$

$$B_{ij,k} = \tfrac{1}{2} Z_{ijk} + \tfrac{1}{2} Z_{kji} - Z_{ikj}$$

$$B_{ij,k} = \tfrac{1}{2} \cancel{Z_{ijk}} + \tfrac{1}{2} \cancel{Z_{ikj}} - \tfrac{1}{2} \cancel{Z_{ikj}} - \tfrac{1}{2} \cancel{Z_{ijk}} = 0 \; ;$$

$$B_{ij,k} = \tfrac{1}{2} \cancel{Z_{ijk}} - \tfrac{1}{2} Z_{ikj} - \tfrac{1}{2} \cancel{Z_{jik}} + \tfrac{1}{2} Z_{jki}$$

$$B_{ij,k} = \tfrac{1}{2} Z_{ijk} - \tfrac{1}{2} Z_{kji} - \tfrac{1}{2} \cancel{Z_{ikj}} + \tfrac{1}{2} Z_{kij}$$

$$B_{ijk} = \tfrac{1}{2} Z_{ijk} - \tfrac{1}{2} Z_{ikj} - \tfrac{1}{2} Z_{ikj} + \tfrac{1}{2} Z_{ijk} \, .$$

$$\boxed{B_{ijk} = Z_{ijk} - Z_{ikj} \, . = B_{ij,k}}$$

$$B_{ij,k} = \tfrac{1}{2} Z_{jki} - \tfrac{1}{2} Z_{ikj}$$

$$\boxed{B_{ij,k} = \tfrac{1}{2} Z_{kji} - \tfrac{1}{2} Z_{kij} = \tfrac{1}{2} B_{kj,i}}$$

$$\boxed{B_{ij,k} = \tfrac{1}{2} B_{ji,k}}$$

$$\cancel{B_{spq} v^s = (Z_{spq} - Z_{sqp}) v^s = \mathcal{A}_{pq}}$$

$$\cancel{\boxed{\mathcal{A}_{pq} = \left(\frac{\partial h_{sp}}{\partial x^q} - \frac{\partial h_{sq}}{\partial x^p}\right) v^s}}$$ — ~~Antisimétrico.~~

$$B_{ps,q} v^s = Z_{psq} v^s - Z_{pqs} v^s$$

$$\mathcal{S}(B_{ps,q} v^s) = \left\{ \begin{aligned} &\tfrac{1}{2} Z_{psq} v^s - \tfrac{1}{2} Z_{pqs} v^s \\ &+ \tfrac{1}{2} Z_{qsp} v^s - \tfrac{1}{2} Z_{qps} v^s \end{aligned} \right\}$$

$$\boxed{\mathcal{S}_{pq} = \tfrac{1}{2} Z_{psq} v^s + \tfrac{1}{2} Z_{qsp} v^s - Z_{pqs} v^s}$$

$$\cancel{\mathcal{A}_{ps,q} v^s} = \left\{ \begin{aligned} &\tfrac{1}{2} Z_{psq} v^s - \tfrac{1}{2} \cancel{Z_{pqs}} v^s \\ &- \tfrac{1}{2} Z_{qsp} v^s + \tfrac{1}{2} \cancel{Z_{qps}} v^s \end{aligned} \right\} = \mathcal{A}_{pq}$$

$$\cancel{\mathcal{A}_{ps,q}\, v}$$

$$\boxed{\mathcal{A}_{pq} = \tfrac{1}{2} Z_{psq} v^s - \tfrac{1}{2} Z_{qsp} v^s}$$

$$B_{sp,q} v^s = Z_{spq} v^s - Z_{sq} v^s$$

$$\boxed{B_{sp,q} v^s = 2\mathcal{A}_{pq}}$$

$$B_{ps,q} v^s = Z_{psq} v^s - Z_{pqs} v^s$$

$$\boxed{B_{ps,q} v^s = \mathcal{S}_{pq} + \mathcal{A}_{pq}}$$

$$B_{pq,s} v^s = Z_{pqs} v^s - Z_{psq} v^s$$

$$\cancel{\mathcal{S}_{pq} t}$$

$$\boxed{B_{pq,s} v^s = -\mathcal{S}_{pq} - \mathcal{A}_{pq}}$$

$$\cancel{B}$$

$$B_{\underset{\smile}{sp},q} v^s = Z_{spq} v^s - \tfrac{1}{2} Z_{sqp} v^s - \tfrac{1}{2} Z_{pqs} v^s$$

$$B_{\underset{\smile}{sp},q} v^s = Z_{psq} v^s - \tfrac{1}{2} Z_{qsp} v^s - \tfrac{1}{2} Z_{pqs} v^s$$

$$\boxed{B_{\{sp\},q} v^s = \tfrac{1}{2} \mathcal{S}_{pq} + \tfrac{3}{2} \mathcal{A}_{pq}}$$

$$\boxed{B_{ps,q} v^s = \tfrac{1}{2} S_{pq} + \tfrac{3}{2} A_{pq}}$$

$$B_{pq,s} v^s = Z_{pqs} v^s - \tfrac{1}{2} Z_{psq} v^s - \tfrac{1}{2} Z_{sqp} v^s$$

$$B_{pq,s} v^s = -\tfrac{1}{2} Z_{psq} v^s - \tfrac{1}{2} Z_{qsp} v^s + Z_{pqs} v^s$$

$$\boxed{B_{pq,s} v^s = - S_{pq}}$$

$$\cancel{B_{spq} v^s = \tfrac{1}{2} Z_{sqp} v^s + \tfrac{1}{2} Z_{psq} v^s - Z_{spq} v^s}$$

$$B_{spq} v^s = \tfrac{1}{2} Z_{spq} v^s + \tfrac{1}{2} Z_{qps} v^s - Z_{sqp} v^s$$

$$-\tfrac{1}{2} S_{pq} + \tfrac{3}{2} A_{pq} =$$

$$-\tfrac{1}{4} Z_{psq} v^s - \tfrac{1}{4} Z_{qsp} v^s + \tfrac{1}{2} Z_{pqs} v^s$$

$$+ \tfrac{3}{4} Z_{psq} v^s - \tfrac{3}{4} Z_{qsp} v^s$$

$$-\tfrac{1}{2} S_{pq} + \tfrac{3}{2} A_{pq} = \tfrac{1}{2} Z_{spq} v^s + \tfrac{1}{2} Z_{pqs} v^s - Z_{sqp} v^s$$

$$\boxed{B_{sp,q}\, v^s = -\frac{1}{2} S_{pq} + \frac{3}{2} A_{pq}}$$

$$B_{ps,q}\, v^s = \frac{1}{2} Z_{psq} v^s + \frac{1}{2} Z_{qsp} v^s - Z_{pqs} v^s$$

$$\boxed{B_{ps,q}\, v^s = S_{pq}}$$

$$B_{pq,s}\, v^s = -\frac{1}{2} S_{qp} + \frac{3}{2} A_{qp}$$

$$\boxed{B_{pq,s}\, v^s = -\frac{1}{2} S_{pq} - \frac{3}{2} A_{pq}}$$

$$\cancel{B_{p}}$$

$$\boxed{B_{sp,q}\, v^s = 0}$$

$$\cancel{B_{spq}}$$

$$\boxed{B_{ps,q}\, v^s = 0}$$

$$\boxed{B_{pq,s}\, v^s = 0}$$

$$B_{sp,q}\, v^s = \frac{1}{2} B_{qp,s}\, v^s$$

$$B_{sp,q}\, v^s = -\frac{1}{2} \mathcal{S}_{qp} - \frac{1}{2} \mathcal{A}_{qp}$$

$$\boxed{B_{sp,q}\, v^s = -\frac{1}{2} \mathcal{S}_{pq} + \frac{1}{2} \mathcal{A}_{pq}}$$

$$\cancel{B_{ps,q}\, v^s =}$$

$$\boxed{B_{ps,q}\, v^s = +\frac{1}{2} \mathcal{S}_{pq} - \frac{1}{2} \mathcal{A}_{pq}}$$

$$\cancel{B_{pq,s}\, v^s = \frac{1}{2} B_{qp,s}}$$

$$B_{pq,s}\, v^s = \frac{1}{2} B_{sqp}\, v^s$$

$$\cancel{B_{pq,s}\, v^s = -\frac{1}{2} \mathcal{S}_{qp} + \frac{3}{2} \mathcal{A}_{qp}}$$

$$\cancel{\boxed{B_{pq,s}\, v^s = -\frac{1}{2} \mathcal{S}_{pq} - \frac{3}{2} \mathcal{A}_{pq}}}$$

$$B_{pq,s} v^s = \mathcal{A}_{qp} = -\mathcal{A}_{pq}$$

$$\boxed{B_{pq,s} v^s = -\mathcal{A}_{pq}}$$

$$B_{sp,q} v^s = \frac{1}{2} B_{psq} v^s$$

$$\boxed{B_{sp,q} = \frac{1}{2} \mathcal{S}_{pq} + \frac{1}{2} \mathcal{A}_{pq}}$$

$$B_{ps,q} = \frac{1}{2} B_{sp,q}$$

$$B_{ps,q} v^s = \frac{1}{2} B_{sp,q} v^s$$

$$\boxed{B_{ps,q} v^s = \mathcal{A}_{pq}}$$

$$B_{pq,s} = \frac{1}{2} B_{qp,s}$$

$$B_{pq,s} v^s = \frac{1}{2} B_{qp,s} v^s = -\frac{1}{2} \mathcal{S}_{pq} + \frac{1}{2} \mathcal{A}_{pq}$$

$$\boxed{B_{pq,s} v^s = -\frac{1}{2} \mathcal{S}_{pq} + \frac{1}{2} \mathcal{A}_{pq}}$$

$$B_{spq} = B_{sp,q}$$

$$\boxed{B_{sp,q}\, v^s = 2\mathcal{A}_{pq}}$$

$$\boxed{B_{ps,q}\, v^s = \mathcal{S}_{pq} + \mathcal{A}_{pq}}$$

$$\boxed{B_{pq,s}\, v^s = -\mathcal{S}_{pq} - \mathcal{A}_{pq}}$$

Conclusión: las relaciones de las páginas 23 y 24 son válidas para el campo general de Birkhoff y no sólo para el campo central!

Tensor $a_i v_j$

$$a_i = \left(\frac{\partial h_{im}}{\partial x^n} - \frac{\partial h_{mn}}{\partial x^i}\right) v^m v^n$$

$$a_i v_j = \left(\frac{\partial h_{im}}{\partial x^n} - \frac{\partial h_{mn}}{\partial x^i}\right) v^m v^n v_j$$

$$a_j v_i = \left(\frac{\partial h_{jm}}{\partial x^n} - \frac{\partial h_{mn}}{\partial x^j}\right) v^m v^n v_i$$

$$\cancel{\frac{d}{ds}\left(x^i v_j\right) = v^i v_j + x^i a_j}$$

$$\frac{d}{ds}\left(v_i v_j\right) = v_i a_j + a_i v_j$$

$$\boxed{\hat{\nu} = \tfrac{1}{2}\hat{v} - \hat{\tau}}$$

$$\hat{\nu}' = \tfrac{1}{2}\hat{a} - \hat{\tau}'$$

$$\alpha' = \hat{\nu}' = \tfrac{1}{2}\hat{a} - \hat{\tau}'$$

$$\alpha_i \nu_j - \alpha_j \nu_i$$

$$\nu_i = \left(-\tfrac{1}{2}t', -\tfrac{1}{2}x', -\tfrac{1}{2}y', -\tfrac{1}{2}z'\right)$$

$$\alpha_i = \left(-\tfrac{1}{2}t'', -\tfrac{1}{2}x'', -\tfrac{1}{2}y'', -\tfrac{1}{2}z''\right)$$

$$\dot{x} = \frac{x'}{t'}$$

$$\boxed{\frac{d}{dt} = \frac{1}{t'}\frac{d}{ds}}$$

$$\ddot{x} = \frac{t'x'' - x't''}{t'^3}$$

$$\ddot{x} = \frac{x''}{t'^2} - \dot{x}\frac{t''}{t'^2}$$

$$\frac{t''}{t'^2} = -\frac{M\dot{r}}{r^2}$$

$$\frac{x''}{t'^2} = + M$$

$$x'' = \frac{Mx}{r^3} - \frac{2Mx}{r^3}t'^2 + \frac{Mx'r'}{r^2}$$

$$1 - 2t'^2 = \begin{cases} t'^2 - x'^2 - y'^2 - z'^2 \\ -2t'^2 \end{cases}$$

$$1 - 2t'^2 = -t'^2 - x'^2 - y'^2 - z'^2$$

$$\frac{x''}{t'^2} = \frac{Mx}{r^3}\left[-1 - \dot{x}^2 - \dot{y}^2 - \dot{z}^2\right] + \frac{M\dot{x}\dot{r}}{r^2}$$

$$\frac{x''}{t'^2} = -\frac{Mx}{r^3} - \frac{Mx}{r^3}v^2 + \frac{M\dot{x}\dot{r}}{r^2}$$

$$\frac{t''}{t'^2} = -\frac{M\dot{r}}{r^2}$$

$$\boxed{\begin{aligned} \ddot{x} &= -\frac{Mx}{r^3} - \frac{Mx}{r^3}v^2 + \frac{2M\dot{x}\dot{r}}{r^2} \\ \ddot{y} &= -\frac{My}{r^3} - \frac{My}{r^3}v^2 + \frac{2M\dot{y}\dot{r}}{r^2} \\ \ddot{z} &= -\frac{Mz}{r^3} - \frac{Mz}{r^3}v^2 + \frac{2M\dot{z}\dot{r}}{r^2} \end{aligned}}$$

$$\vec{a} = (\ddot{x}, \ddot{y}, \ddot{z})$$

$$\vec{v} = (\dot{x}, \dot{y}, \dot{z})$$

$$\boxed{a_\alpha v_\beta - a_\beta v_\alpha}$$

$$\vec{a} = (\ddot{x}, \ddot{y}, \ddot{z})$$

$$\ddot{x} = -\frac{Mx}{r^3} - \frac{Mx}{r^3}v^2 + \frac{2M\dot{x}\dot{r}}{r^2}$$

$$\ddot{y} = -\frac{My}{r^3} - \frac{My}{r^3}v^2 + \frac{2M\dot{y}\dot{r}}{r^2}$$

$$\ddot{z} = -\frac{Mz}{r^3} - \frac{Mz}{r^3}v^2 + \frac{2M\dot{z}\dot{r}}{r^2}$$

$$\vec{v} = (\dot{x}, \dot{y}, \dot{z})$$

$$\boxed{\boxed{[a_1 v_3 - a_3 v_1]}}$$

$$\boxed{\left[a_n v_3 - a_3 v_n\right]}$$

$$\begin{array}{c|c|c}
0 & -\frac{M}{r^3}(1+v^2)(x\dot{y}-\dot{x}y) & -\frac{M}{r^3}(1+v^2)(x\dot{z}-\dot{x}z) \\
\hline
+\frac{M}{r^3}(1+v^2)(x\dot{y}-\dot{x}y) & 0 & -\frac{M}{r^3}(1+v^2)(y\dot{z}-\cancel{\dot{y}}z) \\
\hline
+\frac{M}{r^3}(1+v^2)(x\dot{z}-\dot{x}z) & +\frac{M}{r^3}(1+v^2)(y\dot{z}-\dot{y}z) & 0
\end{array}$$

$$\vec{\nabla}\frac{M}{r} = \left(-\frac{Mx}{r^3}, -\frac{My}{r^3}, -\frac{Mz}{r^3}\right)$$

$$\vec{v} = (\dot{x}, \dot{y}, \dot{z})$$

$$\boxed{\vec{\nabla}_\alpha \frac{M}{r} v_\beta - \vec{\nabla}_\beta \frac{M}{r} v_\alpha}$$

$$\boxed{\vec{\nabla}_\alpha \frac{M}{r} v_\beta - \vec{\nabla}_\beta \frac{M}{r} v_\alpha}$$

0	$-\frac{M}{r^3}(x\dot{y} - \dot{x}y)$	$-\frac{M}{r^3}(x\dot{z} - \cancel{\dot{x}}z)$
$+\frac{M}{r^3}(x\dot{y} - \dot{x}y)$	0	$-\frac{M}{r^3}(y\dot{z} - \dot{y}z)$
$+\frac{M}{r^3}(x\dot{z} - \dot{x}z)$	$+\frac{M}{r^3}(y\dot{z} - \dot{y}z)$	0

$$a_\alpha v_\beta - a_\beta v_\alpha = (1+v^2)\left[\vec{\nabla}_\alpha \frac{M}{r} v_\beta - \vec{\nabla}_\beta \frac{M}{r} v_\alpha\right]$$

$$\vec{\nu} = \cancel{\alpha}\, f(v)\, \vec{v}$$

$$\vec{\alpha} = f'(v)\,\dot{v}\,\vec{v} + f(v)\,\vec{a}$$

$$\vec{\alpha} = \dot{\vec{\nu}}$$

$$\vec{\alpha} \times \vec{\nu} = [f(v)]^2 [\vec{a} \times \vec{v}]$$

$$\cancel{\frac{\vec{\alpha} \times \vec{\nu}}{[f(v)]^2} = (1+v^2)\left[\vec{\nabla}_\alpha \frac{M}{\quad}\right.}$$

$$\frac{\vec{\alpha} \times \vec{\nu}}{[f(v)]^2} = (1+v^2)\left[\vec{\nabla}\frac{M}{r} \times \vec{v}\right]$$

$$\frac{\vec{\alpha} \times \vec{\nu}}{[f(v)]^2} = (1+v^2)\left[\vec{\nabla}\frac{M}{r} \times \frac{\vec{\nu}}{f(v)}\right]$$

$$\cancel{f(v)(1+v^2) = 1}$$

$$\boxed{f(v) = \frac{1}{1+v^2}}$$

$$\boxed{\dot{\vec{\nu}} \times \vec{\nu} = \vec{\nabla}\frac{M}{r} \times \vec{\nu}}$$

$$\vec{\nu} = \frac{\vec{v}}{1+v^2}$$

$$\nu_\alpha \nu_\beta \propto \frac{M x_\alpha}{r} \nu_\beta$$

$$\int_A^B \int_A^P \frac{M}{r} \frac{\dot{x}}{(1+v^2)}$$

$$\int_A^B \int_A^P \frac{M}{r} \frac{\dot{x}}{1+}$$

$$\int_A^B \left\{ \int_A^P \frac{M \, dX + dY + dZ}{R(1+V^2)} \right\} dx$$